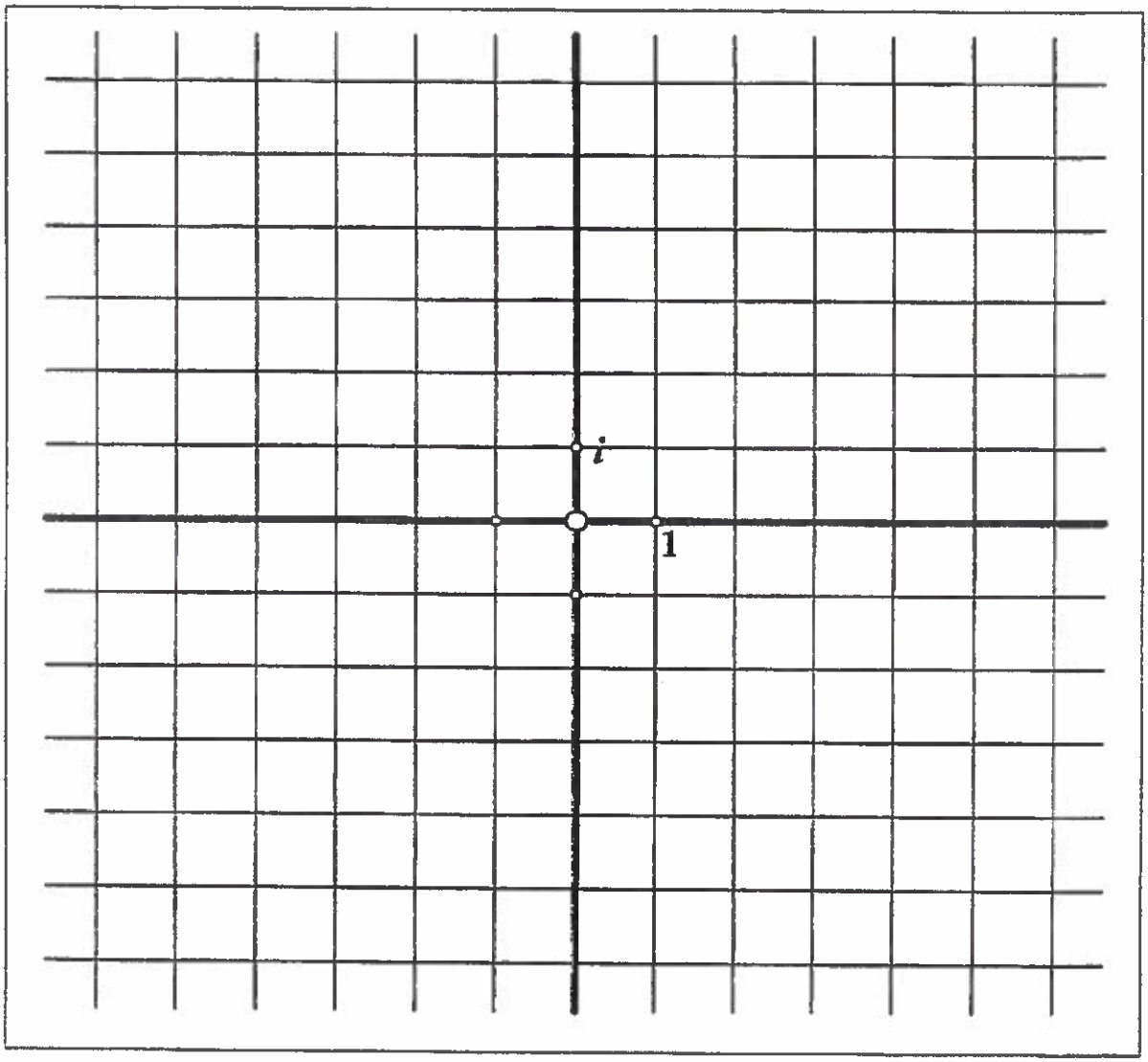
Answers

WORKSHEET #4 TRIGONOMETRIC REPRESENTATION



Plot each of these complex numbers and compute its absolute value and angle of rotation (exact value).

1. $z = 1 + i$ $|z| = \sqrt{2}$ $\theta = \frac{\pi}{4}$

2. $z = 1 - i$ $|z| = \sqrt{2}$ $\theta = \frac{7\pi}{4}$

3. $z = -2 + 2\sqrt{3}i$ $|z| = 4$ $\theta = \frac{2\pi}{3}$

Identify and plot each of the complex with these absolute values and angles.

4. $r = \sqrt{2}$ and $\theta = 135°$ $a = \sqrt{2}\cos 135^\circ = -1$ $b = \sqrt{2}\sin 135^\circ = 1$

5. $r = 2$ and $\theta = \frac{5\pi}{4}$ $a = 2\cos\frac{5\pi}{4} = -\sqrt{2}$ $b = 2\sin\frac{5\pi}{4} = -\sqrt{2}$

6. $z = 6$ and $\theta = \frac{\pi}{6}$ $a = 6\cos\frac{\pi}{6} = 3\sqrt{3}$ $b = 6\sin\frac{\pi}{6} = 3$

WORKSHEET #3 COMPLEX ARITHMETIC II

Express the following quotients in the form a+bi.

1. $\frac{-2+i}{3-2i}$ $(3+2i)$ $(3+2i)$

$\frac{-8-i}{13} = -\frac{8}{13} - \frac{1}{13}i$

$\frac{-6-4i+3i+2i^2}{13}$

2. $\frac{4+3i}{3-4i}$ $(3+4i)$ $(3+4i)$

$\frac{25i}{25} = i$

3. $\frac{1+2i}{1-2i}$ $(1+2i)$ $(1+2i)$

$\frac{-3+4i}{5} = -\frac{3}{5} + \frac{4}{5}i$

4. $\frac{(3-i)(-1+2i)}{2-3i}$ $(2+3i)$

$-3+6i+i-2i = -1+7i$

$= \frac{-23+11i}{13} = -\frac{23}{13} + \frac{11}{13}i$

5. $\frac{7+2i}{3i}$ i i

$\frac{7i+2i^2}{3i^2}$ $\frac{7i-2}{-3}$ $\frac{2-7i}{3}$

$\frac{2}{3} - \frac{7i}{3}$

WORKSHEET #2 COMPLEX ARITHMETIC I

Carry out the following computations:

1. $(4+6i)+(8+7i)$ $12+13i$

2. $(7+i)+(3-i)$ 10

$i^2 = -1$

3. $(3+i)(4i)$ $12i + 4i^2 = -4+12i$

4. $(5+3i)(6-2i)$ $36+8i$

5. $(4+6i)[(7+i)+(3-3i)]$ $52+52i$

$(4+6i)(10-2i) =$

6. $|5+3i| \times |6-2i|$ $(\sqrt{34})(\sqrt{40} = 4\sqrt{85}$

same answers

7. $|(5+3i)(6-2i)|$ $|36+8i| = 4\sqrt{85}$

8. $|a+bi||c+di|$ $\left(\sqrt{a^2+b^2}\right)\left(\sqrt{c^2+d^2}\right) = \sqrt{(a^2+b^2)(c^2+d^2)} = \sqrt{a^2c^2+a^2d^2}$

$b^2c^2+b^2d^2$

9. $|(a+bi)(c+di)|$

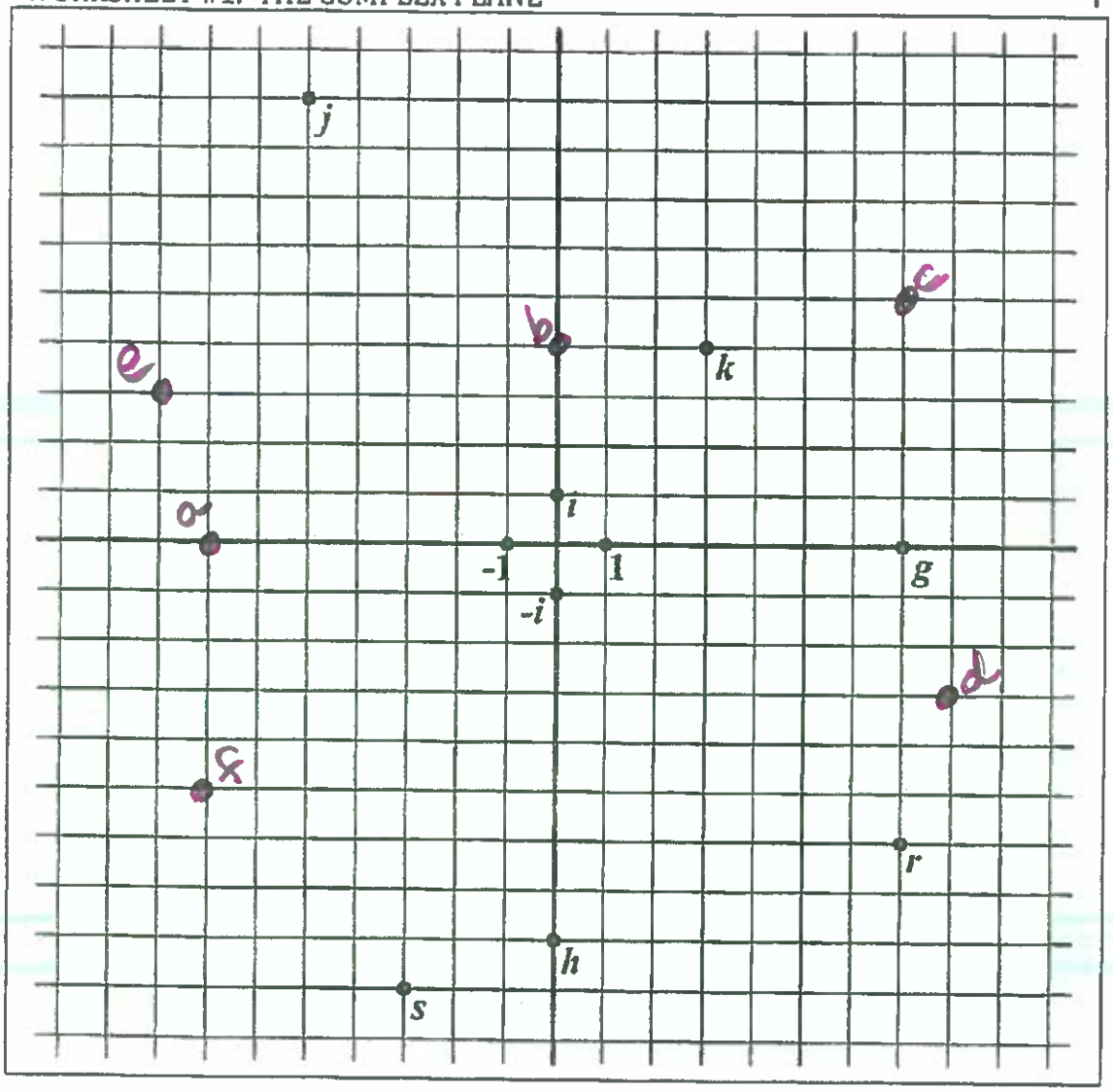
$(ac-bd)+(bc+ad)i \quad \sqrt{(ac-bd)^2-(bc+ad)^2}$

$= \sqrt{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$

Acc. Precalculus

Name: Key

WORKSHEET #1: THE COMPLEX PLANE



Plot the following complex numbers:

a = -7 $|a| = 7$

b = 5*i* $|b| = 5$

c = 7+5*i* $|c| = \sqrt{74}$

d = 8-3*i* $|d| = \sqrt{73}$

e = -8+3*i* $|e| = \sqrt{73}$

f = -7-5*i* $|f| = \sqrt{74}$

Identify the plotted complex numbers:

g = 7 $|g| = |7| = 7$

h = $8i$ $|h| = |8| = 8$

j = $5+9i$ $|j| = \sqrt{106}$

k = $3+4i$ $|k| = 5$

r = $7-6i$ $|r| = \sqrt{85}$

s = $-3-9i$ $|s| = 3\sqrt{10}$

Compute the absolute value of each of these complex numbers. Write these moduli beside the letters above.

$|z| = \sqrt{a^2+b^2}$

Modulus

WORKSHEET #5 COMPLEX NUMBERS IN POLAR AND RECTANGULAR FORM

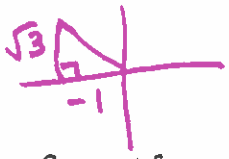
Convert to trigonometric (polar) form. Use exact radian measure.

1. $z_1 = 7 + 7i$

$7\sqrt{2}$ $\theta = \frac{\pi}{4}$ $7\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$7\sqrt{2}\ \text{cis}\ \frac{\pi}{4}$

2. $z_2 = -1 + i\sqrt{3}$



$|z_2| = 2$ $\theta = \frac{2\pi}{3}$

$\sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

$2\cos\frac{2\pi}{3} + 2i\sin\frac{2\pi}{3}$ or $2\ \text{cis}\ \frac{2\pi}{3}$

Convert from polar to rectangular. Give exact values.

3. $z_3 = 6(\cos\pi + i\sin\pi)$

$6\cos\pi + 6i\sin\pi$

$6(-1) + 6i(0) = -6$ $-6 + 0i$

4. $z_4 = \sqrt{7}\ cis\ \frac{5\pi}{6}$

$\sqrt{7}\left(\cos\frac{5\pi}{6}\right) + \sqrt{7}i\left(\sin\frac{5\pi}{6}\right)$

$\sqrt{7}\left(\frac{-\sqrt{3}}{2}\right) + \sqrt{7}i\left(\frac{1}{2}\right) = \frac{\sqrt{21}}{2} + \frac{\sqrt{7}}{2}i$

Using the complex numbers above, find the following using the polar forms of z: (exact values)

5. $z_2 z_3$

$\left(2\ \text{cis}\ \frac{2\pi}{3}\right)\left(6\ \text{cis}\ \pi\right)$ $12\ \text{cis}\left(\frac{5\pi}{3}\right)$

6. $\frac{z_4}{z_1}$

$\frac{\sqrt{7}\ \text{cis}\ \frac{5\pi}{6}}{7\sqrt{2}\ \text{cis}\ \frac{\pi}{4}} = \frac{\sqrt{7}}{7\sqrt{2}}\ \text{cis}\left(\frac{5\pi}{6} - \frac{\pi}{4}\right) =$

$\frac{\sqrt{14}}{14}\ \text{cis}\left(\frac{2\pi}{12}\right)$